Measuring Length and Area

Geometry

Chapter 11

Geometry 11

This Slideshow was developed to accompany the textbook

- *Big Ideas Geometry*
- *By Larson and Boswell*
- *2022 K12 (National Geographic/Cengage)*

Some examples and diagrams are taken from the textbook.

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After this lesson…

- *I can find the perimeter of rectangles and triangles.*
- *I can find the area of rectangles and triangles.*

11.1 Areas of Triangles and Parallelograms (1.4)

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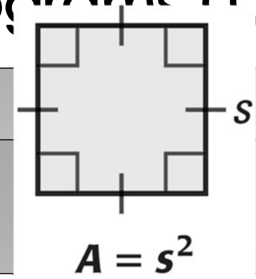


Area of a Square

$$A = s^2$$

Where s is the length of a side.

Area Congruence Postulate

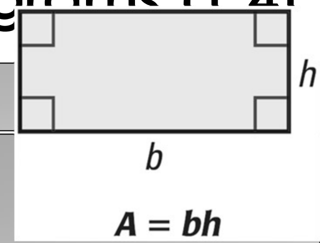
If 2 polygons are congruent, then they have the same area.

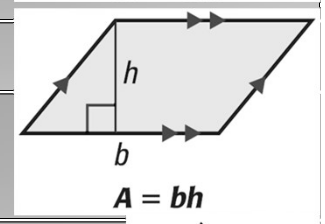
Area Addition Postulate

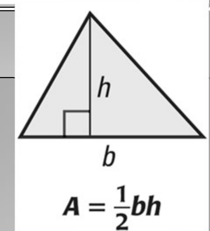
The total area is the sum of the areas of the nonoverlapping parts.

11.1 Areas of Triangles and Parallelograms (1.4)

Area of a Rectangle
$A = bh$ Where b is the base and h is the height.
Area of a Parallelogram
$A = bh$ Where b is the base and h is the height.
Area of a Triangle
$A = \frac{1}{2}bh$ Where b is the base and h is the height.







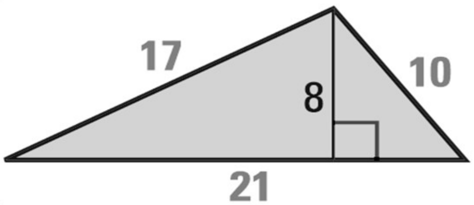
Rectangle can be divided into b by h unit squares.

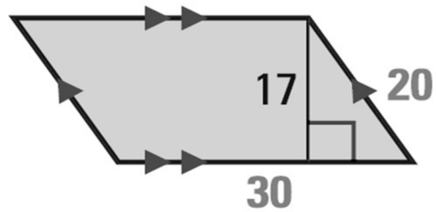
Parallelogram can be cut apart and built into a rectangle.

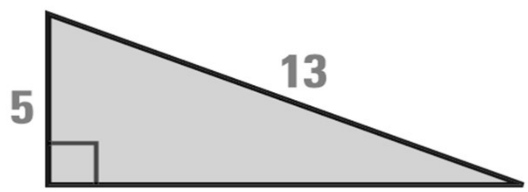
Triangle is ½ a parallelogram.

11.1 Areas of Triangles and Parallelograms (1.4)

Find the perimeter and area of the polygon.







$$P = 17 + 10 + 21 = 48$$
$$A = \frac{1}{2}(21)(8) = 84$$

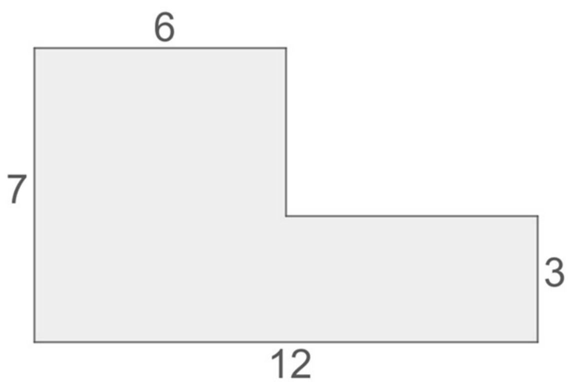
$$P = 20 + 30 + 20 + 30 = 100$$
$$A = 30(17) = 510$$

$$a^2 + b^2 = c^2$$
$$5^2 + b^2 = 13^2$$
$$25 + b^2 = 169$$
$$b^2 = 144$$
$$b = 12$$
$$P = 5 + 12 + 13 = 30$$
$$A = \frac{1}{2}(5)(12) = 30$$

11.1 Areas of Triangles and Parallelograms (1.4)

- A parallelogram has an area of 153 in^2 and a height of 17 in. What is the length of the base?

- Find the area.



$$A = bh$$
$$153 = b17$$
$$b = 9$$

Length of right rectangle is 6

$$A = 7(6) + 3(6) = 60$$

After this lesson…

- *I can find the area of trapezoids.*
- *I can find the area of rhombuses.*
- *I can find the area of kites.*

11.2 Areas of Trapezoids, Rhombuses, and Kites (11.3)

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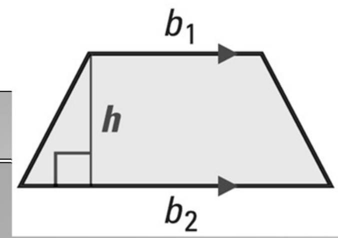
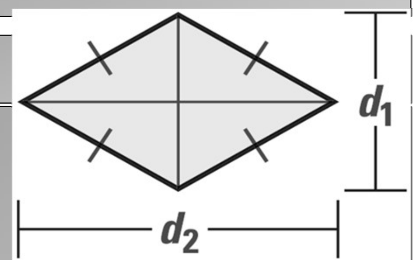


Area of a Trapezoid

$$A = \frac{1}{2}h(b_1 + b_2)$$

Where h is the height and b_1 and b_2 are the bases.



Area of a Rhombus

$$A = \frac{1}{2}d_1 d_2$$

Where d_1 and d_2 are the diagonals.

Trapezoid is a triangle + parallelogram

$$A = \frac{1}{2}(b_2 - b_1)h + b_1 h$$
$$A = \frac{1}{2}b_2 h - \frac{1}{2}b_1 h + b_1 h$$
$$A = \frac{1}{2}b_2 h + \frac{1}{2}b_1 h$$
$$A = \frac{1}{2}h(b_1 + b_2)$$

Rhombus is four small triangles

$$A = 4\left(\frac{1}{2}\left(\frac{1}{2}d_1\right)\left(\frac{1}{2}d_2\right)\right)$$
$$A = \frac{4}{8}d_1 d_2$$

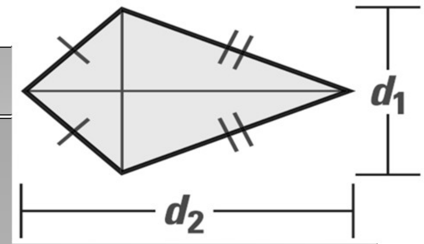
$$A = \frac{1}{2} d_1 d_2$$

11.2 Areas of Trapezoids, Rhombuses, and Kites (11.3)

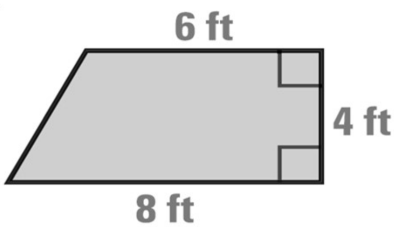
Area of a Kite

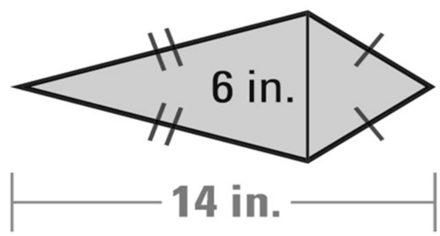
$$A = \frac{1}{2} d_1 d_2$$

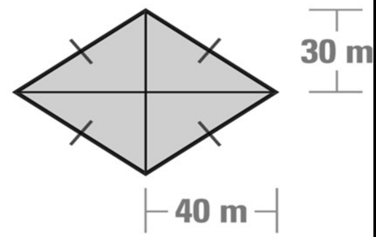
Where d_1 and d_2 are the diagonals.



Find the area







A Kite is two triangles with base d_2 and height ½ d_1

$$A = 2\left(\frac{1}{2}(d_2)\left(\frac{1}{2}d_1\right)\right)$$

$$A = \frac{2}{4} d_1 d_2$$

$$A = \frac{1}{2} d_1 d_2$$

Trapezoid

$$A = \frac{1}{2} 4(6 + 8) = 28$$

Kite

$$A = \frac{1}{2}(6)(14) = 42$$

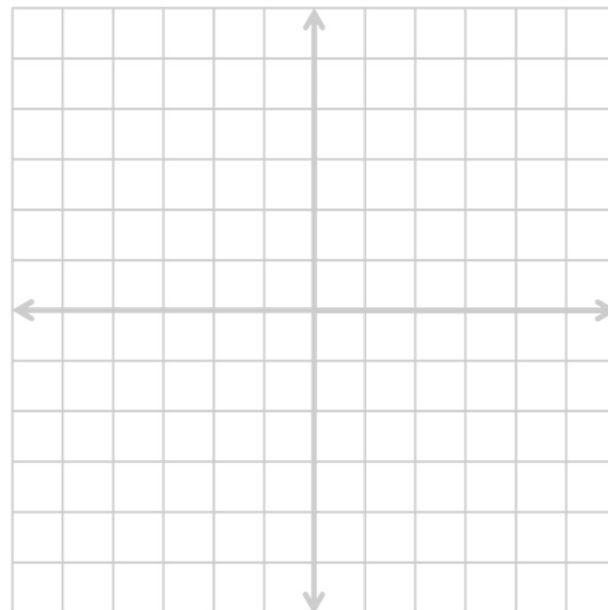
Rhombus

$$A = \frac{1}{2}(80)(60) = 2400$$

11.2 Areas of Trapezoids, Rhombuses, and Kites (11.3)

The area of a kite is 80 ft². One diagonal is 4 times as long as the other. Find the diagonal lengths.

Find the area of a rhombus with vertices $M(1, 3)$, $N(5, 5)$, $P(9, 3)$ and $Q(5, 1)$.



Kite

$$A = \frac{1}{2} d_1 d_2$$
$$d_1 = 4d_2$$
$$80 = \frac{1}{2} 4d_2(d_2)$$
$$80 = 2d_2^2$$
$$40 = (d_2)^2$$
$$d_2 = 2\sqrt{10}$$
$$d_1 = 8\sqrt{10}$$

Rhombus

Diagonals are $d_1 = 8, d_2 = 4$

$$A = \frac{1}{2}(8)(4) = 16$$

After this lesson…

- *I can use the circumference of a circle to find measures.*
- *I can find arc lengths and use arc lengths to find measures.*

11.3 Circumference and Arc Length (11.1)

11.3 Circumference and Arc Le

Circumference of a Circle

- Distance around the circle
- Like perimeter

π

- Ratio of the circumference to the diameter of a circle
- Estimated in 2 Chronicles 4:2 and 1 Kings 7:23 as 3
- 3.141592654…

$$C = \pi d$$
$$C = 2\pi r$$

11.3 Circumference and Arc Length (11.1)

Find the circumference of a circle with diameter 5 inches.

Find the diameter of a circle with circumference 17 feet.

A car tire has a diameter of 28 inches. How many revolutions does the tire make while traveling 500 feet?

$$C = \pi d$$
$$C = \pi 5 = 15.7\ in$$

$$17 = \pi d$$
$$\frac{17}{\pi} = d$$
$$d = 5.41\ ft$$

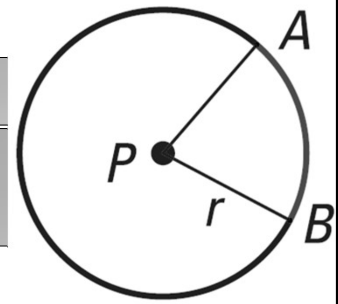
$$28\ in = 2\frac{1}{3} ft$$
$$C = \pi\left(2\frac{1}{3}\right) = 7.3304\ ft$$
$$Revoultions = \frac{500}{7.3304} = 68.2\ rev$$

11.3 Circumference and Arc Length (11.1)

Arc Length

- Portion of the circumference that an arc covers

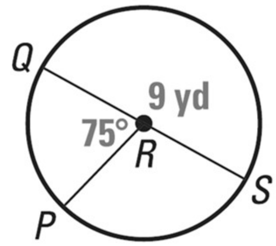
Arc Length
$$\text{Arc Length} = \frac{\text{Arc Measure}}{360°} \cdot 2\pi r$$



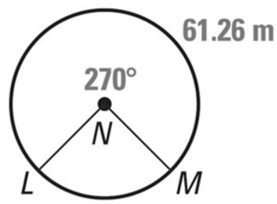
$$Arc\ Length\ \ \widehat{AB} = \frac{m\,\widehat{AB}}{360°} \cdot 2\pi r$$

11.3 Circumference and Arc Length (11.1)

Find the length of $\widehat{PQ}$.



Find the Circumference of $\odot N$.



$$r = 4.5\ yd$$

$$Arc\ Length\ \ \widehat{PQ} = \frac{75}{360} \cdot 2\pi 4.5\ yd = 5.89\ yd$$

$$Arc\ Length\ = 61.26\ m$$

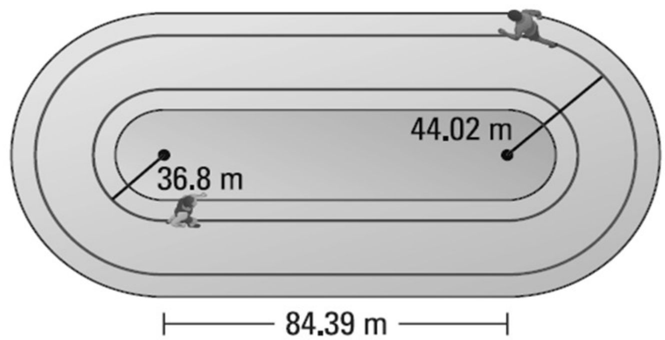
$$61.26\ m = \frac{270}{360} \cdot 2\pi r$$

$$61.26\ m = .75 \cdot 2\pi r$$

$$81.7\ m = 2\pi r = C$$

11.3 Circumference and Arc Length (11.1)

How far does the runner on the blue path travel in one lap. Round to the nearest tenth of a meter.



The two ends make a circle

$$C = 2\pi 44.02\ m = 276.59\ m$$

Add the two straight stretches

$$276.59\ m + 2(84.39\ m) = 445.4\ m$$

After this lesson…

- *I can use the formula for area of a circle.*
- *I can find areas of circles and sectors.*
- *I can solve problems involving areas of sectors.*

11.4 Areas of Circles and Sectors (11.2)

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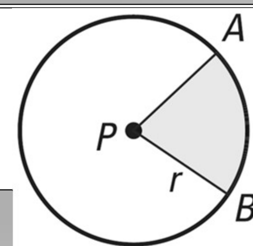
Area of a Circle

$$A = \pi r^2$$

Sector of a Circle

- Fraction of a Circle

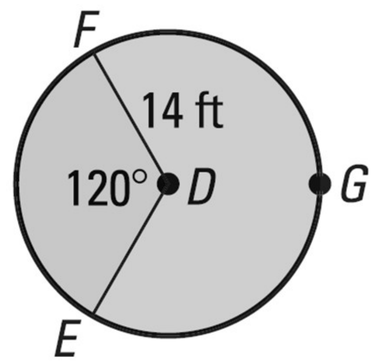


Area of a Sector

$$A = \frac{\text{Arc Measure}}{360°} \cdot \pi r^2$$

11.4 Areas of Circles and Sectors (11.2)

Find area of $\odot D$



Find area of red sector

Find area of blue sector

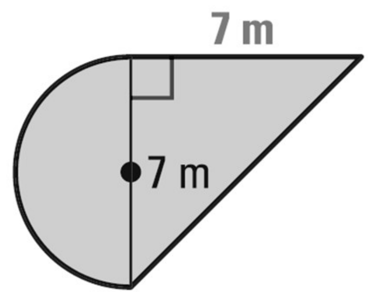
$$A = \pi 14^2 = 615.8\ ft^2$$

$$A = \frac{120}{360}\pi 14^2 = 205.3\ ft^2$$

$$A = \frac{240}{360}\pi 14^2 = 410.5\ ft^2$$

11.4 Areas of Circles and Sectors (11.2)

Find the area of the figure.



Semicircle

$$A = \frac{1}{2}(\pi 3.5^2) = 19.2423\ m^2$$

Triangle

$$A = \frac{1}{2}(7)(7) = 24.5\ m^2$$

Total

$$19.2423\ m^2 + 24.5\ m^2 = 43.7\ m^2$$

After this lesson…

- *I can find the angle measures in regular polygons.*
- *I can find areas of regular polygons.*

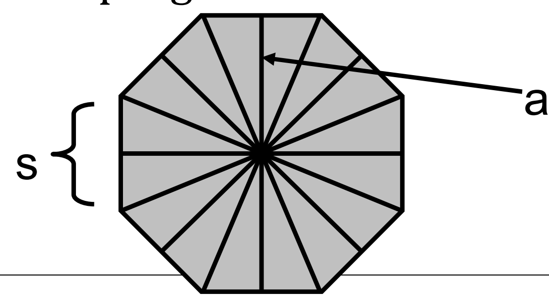
11.5 Areas of Regular Polygons (11.3)

11.5 Areas of Regular Polygons (11.3)

Now that we know how to find the area of a triangle we can find the area of any polygon since it can be broken up into triangles.

For example find the area of a stop sign.

$A = \frac{1}{2}Pa$



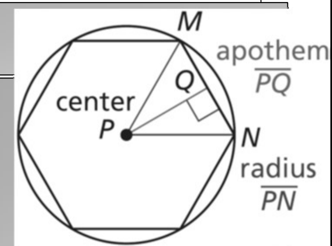
It has 8 sides, I'll call them *s*.
If we draw lines connecting opposite vertices, we have 8 identical triangles.
Draw the altitudes from the center of the sign and call it *a*.
The area of each triangle is ½ *sa*.
The area of the sign then is 8(½ *sa*).
But the perimeter, *P*, is 8*s,* so the Area = ½ *Pa*.

11.5 Areas of Regular Polygons (11.3)

Apothem

- A segment drawn from the center of a regular polygon perpendicular to the edge (also bisects edge)

Area of a Regular Polygon



$$A = \frac{1}{2}Pa$$

Where P is the perimeter and a is the apothem

11.5 Areas of Regular Polygons (11.3)

Typical steps to find area of regular polygon

Find ½ of central angle

- $\frac{1}{2}\left(\frac{360}{n}\right)$

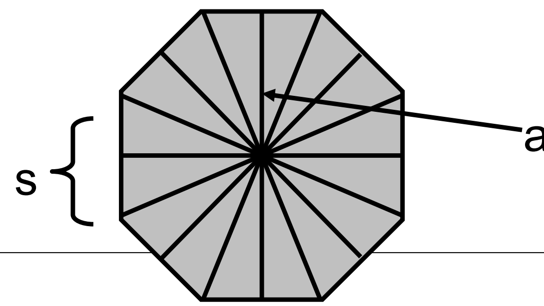
Use trigonometry to find apothem

- tan, sin, cos

Find perimeter

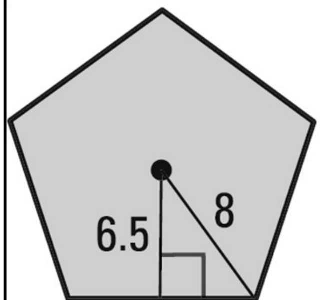
- $P = ns$

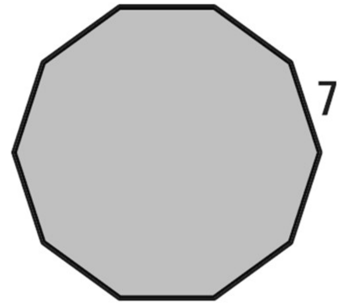
$A = \frac{1}{2}Pa$



11.5 Areas of Regular Polygons (11.3)

- Find the area of the regular polygon.





Pentagon

- Pythagorean theorem to find side

$$6.5^2 + x^2 = 8^2$$
$$42.25 + x^2 = 64$$
$$x^2 = 21.75$$
$$x = \frac{\sqrt{87}}{2} = 4.6637$$
$$s = 2x = 9.3274$$

- Area

$$A = \frac{1}{2}(9.3274 \cdot 5)(6.5) = 151.6$$

Decagon

- Find ½ central angle

$$\frac{1}{2}\left(\frac{360}{10}\right) = 18°$$

- Find apothem

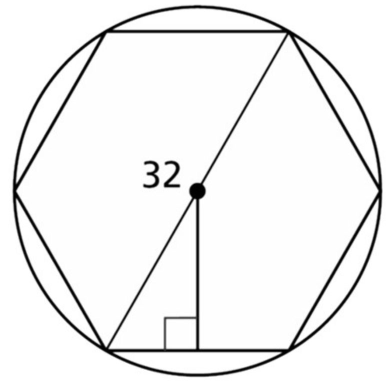
$$\tan 18° = \frac{3.5}{a}$$
$$a \cdot \tan 18° = 3.5$$
$$a = 10.7719$$

- Find area

$$A = \frac{1}{2}(7 \cdot 10)(10.7719) = 377.0$$

11.5 Areas of Regular Polygons (11.3)

A regular hexagon is inscribed in a circle with a diameter of 32 units. Find the area of the hexagon.



Radius is 16

½ central angle

$$\frac{1}{2}\left(\frac{360°}{6}\right) = 30°$$

Apothem

$$\cos 30° = \frac{a}{16}$$
$$a = 16\cos 30° = 8\sqrt{3}$$

Side length

$$\sin 30° = \frac{x}{16}$$
$$x = 16\sin 30° = 8$$
$$s = 2x = 16$$

Perimeter

$$P = 6s = 6(16) = 96$$

Area

$$Area = \frac{1}{2}Pa$$

$$Area = \frac{1}{2}(96)\left(8\sqrt{3}\right) = 665.11$$

After this lesson…

- *I can find probabilities involving segments.*
- *I can find probabilities involving areas.*

11.6 Use Geometric Probability

11.6 Use Geometric Probability

Let's say you are listening to a radio contest where you hear a song and call in and name it.

- The song was supposed to be played between 12:00 and 1:00, but you can only listen from 12:20 to 1:00 because that is when you get out of class.
- What is the probability that you will hear the song?

$$\text{Probability} = \frac{\text{Favorable Outcomes}}{\text{Total Outcomes}}$$

- But we have basically a line (timeline), so Probability will be $\frac{40\ min}{60\ min} = \frac{2}{3} \approx 67\%$

11.6 Use Geometric Probability

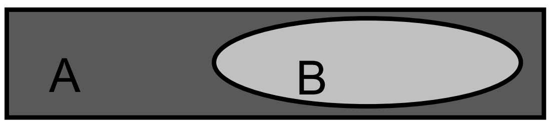
Length Probability Postulate

If a point on AB is chosen at random and C is between A and B, then the probability that the point is on AC is (Length of AC)/(Length of AB).

$$P(AC) = \frac{AC}{AB}$$

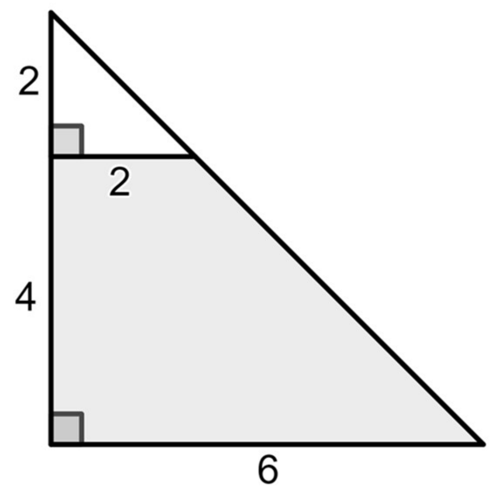
11.6 Use Geometric Probability

Area Probability Postulate
If a point in region A is chosen at random, then the probability that the point is in region B, which is in the interior of region A, is (Area of region B) / (Area of region A) $$P(B) = \frac{Area\ of\ B}{Area\ of\ A}$$



11.6 Use Geometric Probability

Find the probability that a random point is in the shaded region.



$$P(shaded) = \frac{trapezoid}{big\ triangle}$$

$$= \frac{\frac{1}{2}h(b_1 + b_2)}{\frac{1}{2}bh}$$

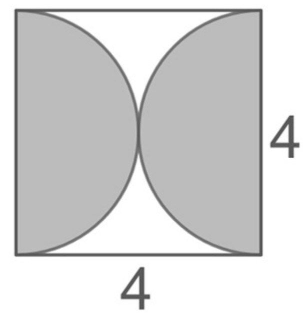
$$= \frac{\frac{1}{2}(4)(2+6)}{\frac{1}{2}(6)(6)}$$

$$= \frac{16}{18}$$

$$= \frac{8}{9} \approx 88.9\%$$

11.6 Use Geometric Probability

Find the probability that a random point is in the shaded region.



$$P(shaded) = \frac{2\ semicircles}{square}$$

$$= \frac{\left(2 \cdot \frac{1}{2}\pi r^2\right)}{s^2}$$

$$= \frac{\pi r^2}{s^2}$$

$$= \frac{\pi(2)^2}{4^2} = \frac{\pi}{4} \approx 0.7854 \approx 78.54\%$$